Let n = problem size and p = the number of processors. There are two contributions to the execution time of a calculation. The first, $\sigma(n)$, is the time for the sequential or serial part of the calculation. The other contribution is $\phi(n)$ which represents the time it takes to evaluate a part of the problem that can be parallelized. The total time for a calculation on a single computer is:

$$t_s = \sigma(n) + \phi(n) \quad .$$

The same calculation on p processors ignoring communication will be,

$$t_p \geq \sigma(n) + \phi(n)/p \quad .$$

Then the speedup, $S(n,p)$, can be defined as,

$$S(n,p) \leq \frac{\sigma(n) + \phi(n)}{\sigma(n) + \phi(n)/p}$$

Let f be the fraction of the calculation that can only be computed serially in the sequential calculation,

$$f = \frac{\sigma(n)}{\sigma(n) + \phi(n)}$$

and solve for $\phi(n)$,

$$\phi(n) = \sigma(n)(\frac{1}{f} - 1) \quad .$$

The new equation for $\phi(n)$ can be substituted into the equation for $S(n,p)$,

$$\begin{aligned} S(n,p) &\leq \frac{\sigma(n) + \sigma(n)(\frac{1}{f} - 1)}{\sigma(n) + \sigma(n)(\frac{1}{f} - 1)/p} \\ &\leq \frac{\frac{1}{f}}{1 + (\frac{1}{f} - 1)/p} \\ &\leq \frac{1}{f + (1-f)/p} \end{aligned}$$

which is Amdahl's Law.

To derive Gustafson's Law, the focus is shifted to the parallel calculation. Let s be the fraction of the calculation that must be calculated serially in a parallel calculation,

$$s = \frac{\sigma(n)}{\sigma(n) + \phi(n)/p}$$

and the fraction that can be parallelized is,

$$\begin{aligned}(1-s) &= \frac{\sigma(n)+\phi(n)/p-\sigma(n)}{\sigma(n)+\phi(n)/p} \\ &= \frac{\phi(n)/p}{\sigma(n)+\phi(n)/p} \quad .\end{aligned}$$

The definitions of $\sigma(n)$ and $\phi(n)$ can be rewritten in terms of s,

$$\sigma(n) = (\sigma(n)+\phi(n)/p)s$$

and

$$\phi(n) = (\sigma(n)+\phi(n)/p)(1-s)p \quad .$$

Using these definitions in the equation for $S(n,p)$, we obtain,

$$\begin{aligned}S(n,p) &\leq \frac{(\sigma(n)+\phi(n)/p)s+(\sigma(n)+\phi(n)/p)(1-s)p}{\sigma(n)+\phi(n)/p} \\ &\leq \frac{(\sigma(n)+\phi(n)/p)(s+(1-s)p)}{\sigma(n)+\phi(n)/p} \\ &\leq s+(1-s)p \\ &\leq p+(1-p)s \quad .\end{aligned}$$

The latter equation is Gustafson's Law.