How to Prove Things

Let P and Q be two compound propositions.

1 Proving Logical Equivalences

syntax:

$$P \equiv Q \quad \text{or} \quad P \Leftrightarrow Q$$

1.1 Using Truth Tables

1. Construct a two truth tables one for P and one for Q having the same order of truth values for the sub-propositions. Then show that the truth tables are the same, that is, the last columns of each table are the same.
2. Construct one truth table for
$$P \leftrightarrow Q$$
and show that $P \leftrightarrow Q$ is a tautology, i.e., the last column of the truth table contains only the T.

1.2 Using Known Logical Equivalences

1. Starting from P, using known logical equivalences (from the tables that were handed out or from the book p. 24), derive Q.
2. Show that $P \leftrightarrow Q$ is a tautology, i.e.,
$$P \leftrightarrow Q \equiv T$$
Starting from $P \leftrightarrow Q$, using known logical equivalences (from the tables that were handed out or from the book p. 24), derive T.

Recommended method: 1.2.1 as it is the least time consuming.

Example (from h.w p.27 q.24): Show that

$$\neg p \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$$

$$\begin{array}{rll}
\neg p \rightarrow (q \rightarrow r) \equiv & \neg p \rightarrow (\neg q \vee r) & \text{by } u \rightarrow v \equiv \neg u \vee v \\
\equiv & p \vee (\neg q \vee r) & \text{by } u \rightarrow v \equiv \neg u \vee v \\
\equiv & (p \vee \neg q) \vee r & \text{by associative law} \\
\equiv & (\neg q \vee p) \vee r & \text{by commutative law} \\
\equiv & \neg q \vee (p \vee r) & \text{by associative law} \\
\equiv & q \rightarrow (p \vee r) & \text{by } \neg u \vee v \equiv u \rightarrow v
\end{array}$$

2 Proving Theorems/Validity of Mathematical Arguments

syntax:

$$\frac{P}{\therefore Q} \quad \text{or} \quad P \Rightarrow Q$$

2.1 Using Rules of Inference

Starting from the premise/premises P derive Q using rules of inference such as *modus ponens* and *syllogism* (from the tables that were handed out or from the book p. 58).

2.2 Proving a Tautology

Show that $p \rightarrow q$ is a tautology, i.e.

$$p \rightarrow q \equiv T$$

in which case you have to prove a logical equivalence using the methods for proving logical equivalences (section 1).

Recommended method: 2.1 as it is the least time consuming.

Example : Show that

$$(p \rightarrow r) \wedge (r \rightarrow s) \wedge (t \vee \neg s) \wedge (\neg t \vee v) \wedge \neg v \Rightarrow \neg p$$

	steps	reason
1.	$p \rightarrow r$	premise
2.	$r \rightarrow s$	premise
3.	$p \rightarrow s$	steps 1,2 and *hypothetical syllogism*
4.	$t \vee \neg s$	premise
5.	$s \rightarrow t$	$t \vee \neg s \equiv \neg s \vee t \equiv s \rightarrow t$
6.	$\neg t \vee v$	premise
7.	$t \rightarrow v$	$\neg t \vee v \equiv t \rightarrow v$
8.	$s \rightarrow v$	steps 5,7 and *hypothetical syllogism*
9.	$p \rightarrow v$	steps 3,8 and *hypothetical syllogism*
10.	$\neg v$	premise
11.	$\neg p$	steps 9,10 and *modus tollens*